DEHN TWISTS ABOUT CURVES ON A SURFACE

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Motivation

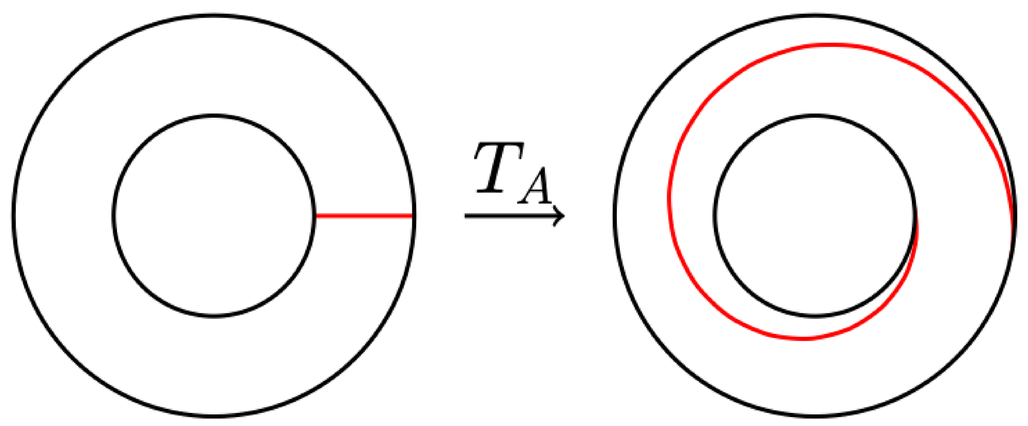
A homeomorphism between surfaces (or any two topological spaces) is a continuous bijection with a continuous inverse. Some of the simplest examples to visualize are rotations, reflections and hyperelliptic involutions. A special homeomorphism that cannot be realized by rigid motions is a Dehn Twist. We first define a map for Dehn twists via annuli. Then we consider simple closed curves on an orientable surface upon realizing that every simple closed curve on an orientable surface is the core of some annulus. We can then fix every point outside of the interior of the choice of annulus and obtain a homeomorphism of the surface, and finally show that the mapping class does not depend on the choice of annulus.

Keywords: homeomorphism, Dehn twist, simple closed curve, mapping class.

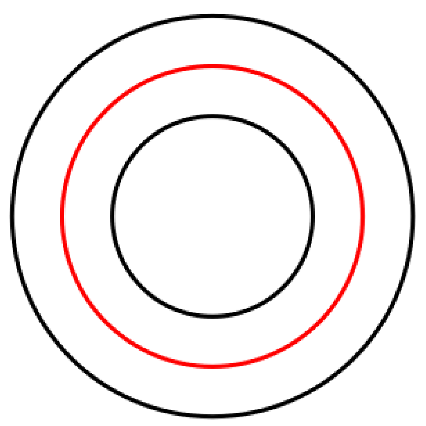
A Dehn Twist on an Annulus

Using polar coordinates (r, θ) for points in the plane $\mathbb{R}^2$, we consider the annulus A made up of those points with $1 \leq r \leq 2$. Then we can define a map

$$\begin{aligned} T_A : A &\longrightarrow A \\ (r, \theta) &\longmapsto (r, \theta - 2\pi r). \end{aligned}$$



Note that the core of an annulus A is the set of points where $r = \frac{3}{2}$.

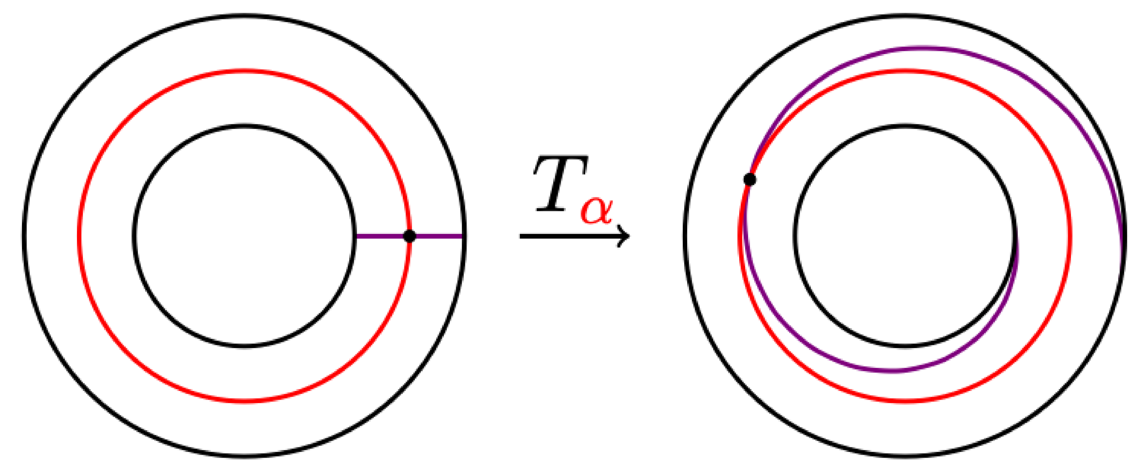


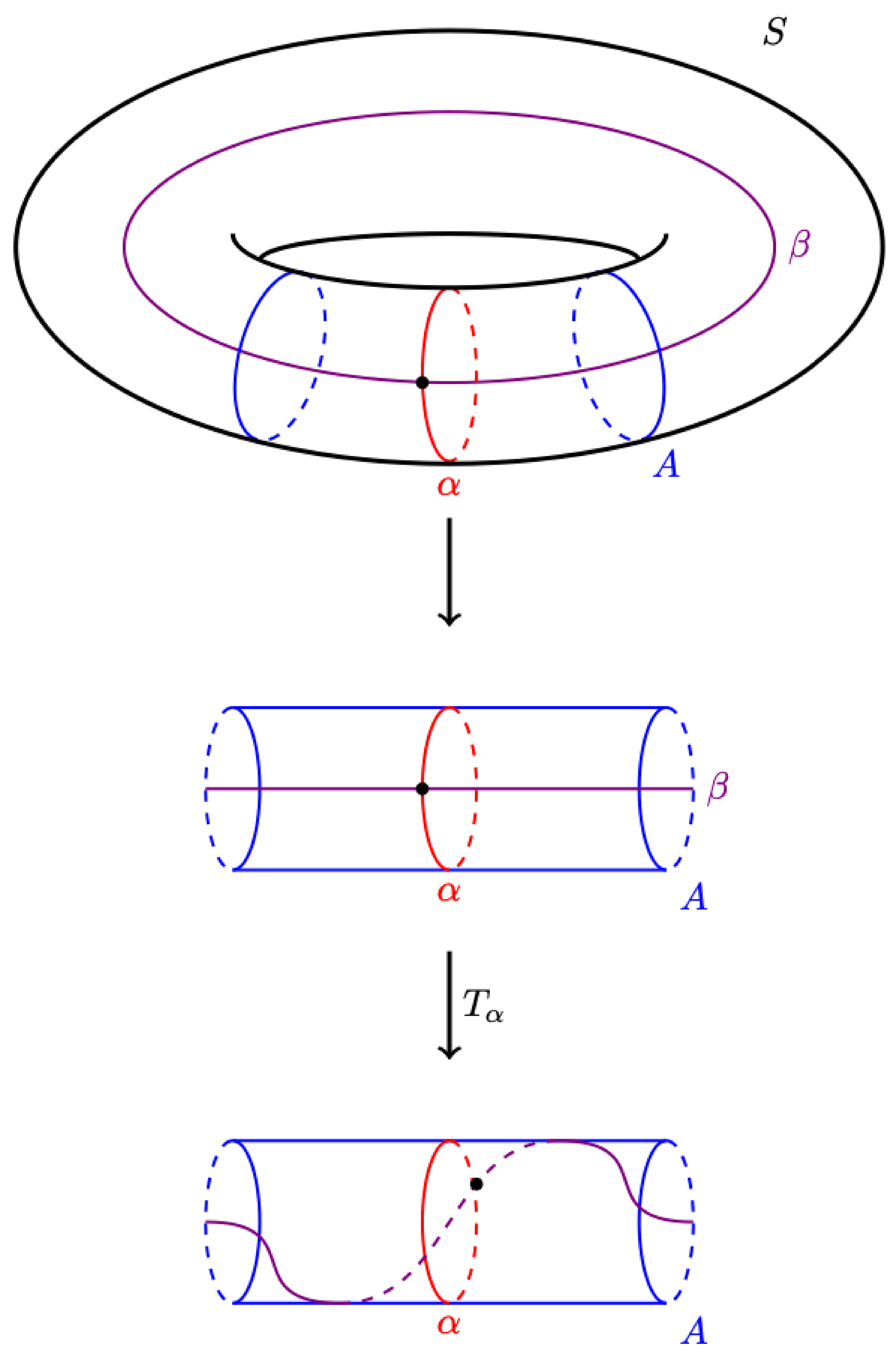
Mapping Class of a Dehn Twist on a Surface

Let α denote the red curve and β the violet arc on an annulus A. Then consider a Dehn twist about α. The image of β under T_α is shown to the right. Notice that the boundary points of the arc are preserved.

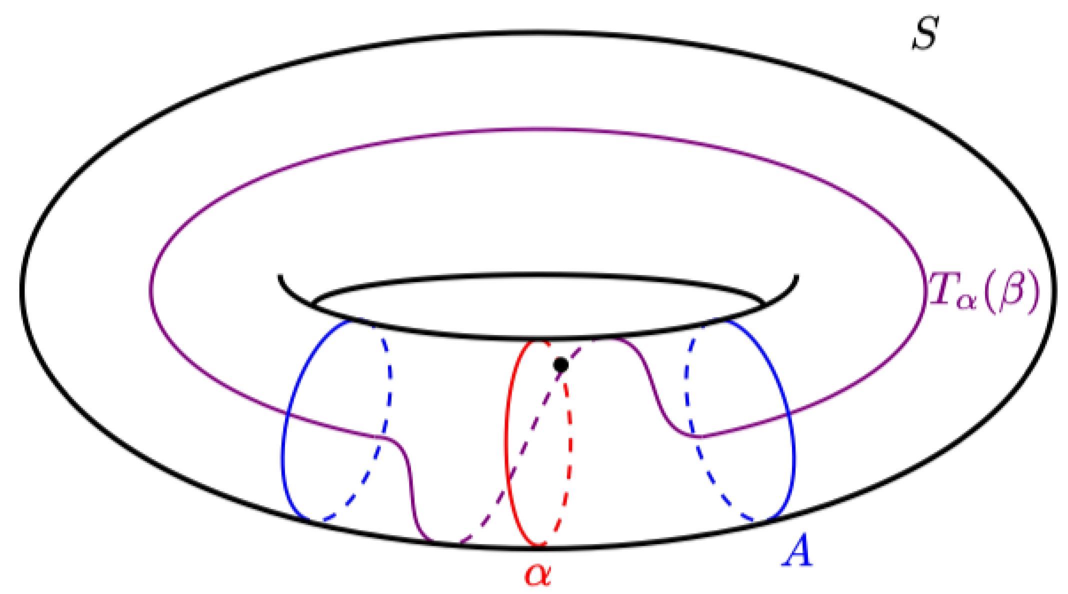
We can extend this notion to Dehn twists about curves on a surface since we can realize every simple closed curve on an orientable surface as the core of some annulus.

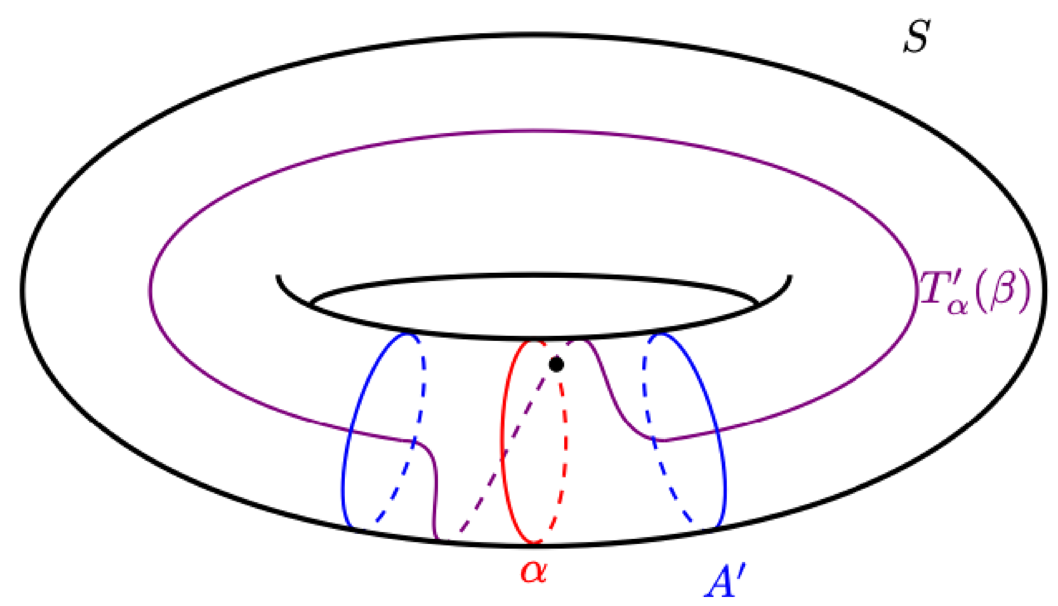
Let S be an orientable surface with two simple closed curves, namely α and β. Then the Dehn twist on S about α is obtained by choosing an annulus A, applying T_α to A and fixing every point in $S \setminus A$, as shown below.





Suppose we choose a different annulus A'. Then we will obtain a curve $T'_\alpha(\beta)$ that is isotopic to $T_\alpha(\beta)$. Moreover, we have that T_α and T'_α belong to the same mapping class and any choice of annulus will yield an element of the mapping class of T_α.





Future Research

- Mapping class groups
- Generating the mapping class group by Dehn twists
- Other generating sets for the mapping class group

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